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A CTMC-Based Parameter Estimation and Optimal Control Framework for Covid-19 Transmission

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Abstract

Covid-19 is a disease that infects the respiratory tract caused a corona virus. This disease has infected human populations around the world which causing a pandemic. The Covid-19 pandemic has become the focus of research on lately. This study aims to construct the latest mathematical model related to the transmission of Covid-19 in the human population. The model that is formed is constructed by considering several epidemiological parameters that are very identical to the actual conditions. In the formation of the model there are many unknown parameters. Therefore, the maximum likelihood method is used to estimate the parameters of the model that is formed. At the start the Covid-19 epidemic model was constructed without control with given assumptions and based on the facts obtained. Then from the model would formed equilibrium point, and basic reproduction number. The next discussion is to designed an optimal control using the Pontryagin Minimum Principle which is applied to reduce the number of people Covid-19 infected by using the backward-forward sweep algorithm. The results of the numerical simulation will later produce an optimal control strategy that is suitable to prevent Covid-19 to be more effective.

1. Introduction

Covid-19 is a disease that infects the respiratory tract caused a corona virus. This disease has infected human populations around the world which causing a pandemic. This phenomenon makes all aspects take action to find the best treatment options and anticipatory ways to prevent this pandemic on a regular basis. From a mathematical perspective, these problems are closely related to the construction and implementation of mathematical models to identify potential solutions. Researchers have studied the coronavirus epidemic in a limited population involving isolation classes [1], involves mobilizing to push for diagnosis and researching how many vaccines are needed to suppress the spread of the Covid-19 pandemic [2], [3]. SIR (Susceptible,

Infected, Recovered) model which modified presented in previous research [4], [5] to project the actual number of infected cases and the specific burden on Covid-19 isolation classes and incentive care units. Several researchers developed a Covid-19 model and studied its dynamic behavior [6], [7], [8].
This study aims to construct the latest mathematical model related to the transmission of Covid-19 in the human population. The model that is formed is constructed by considering several epidemiological parameters that are very identical to the actual conditions. In the formation of the model there are many unknown parameters. Therefore, the maximum likelihood method is used to estimate the parameters of the model that is formed [9]. At the start the Covid-19 epidemic model was constructed without control with given assumptions and based on the facts obtained. Then from the model would formed equilibrium point, and basic reproduction number. The next discussion is to designed an optimal control using the Pontryagin Minimum Principle which is applied to reduce the number of people Covid-19 infected by using the backward-forward sweep algorithm. The results of the numerical simulation will later produce an optimal control strategy that is suitable to prevent Covid-19 to be more effective.

2. Method

2.1 Mathematical Model

This chapter discusses the formulation of a mathematical model of Covid-19 by dividing the human population into five categories based on their health status: susceptible individuals ($\mathrm{S_h}$), exposed/latent individuals ($\mathrm{E_h}$), infected individuals($\mathrm{I_h}$), hospitalized individuals ($\mathrm{H_h}$), recovered individuals (R_h). To formulate the covid 19 model, several limitations or assumptions are given, namely: human birth rate is not constant, there is a process of birth and death, every human being has the same chance of contracting Covid-19, pure death (not due to covid-19) occurred in all classes, and deaths from Covid-19 only occurred within a class of infected individuals. The variables and parameters used in the Covid-19 mathematical model that will be formulated can be seen in Table 1 and Table 2 as follows:

Table 1 *Description of Covid-19 Mathematical Model Variables*

Variable	Information
$S_h(t)$	The number of humans who are susceptible to time t
$E_h(t)$	Number of humans latent at time t
$I_h(t)$	The number of people Covid-19 infected at that time t
$H_h(t)$	Number of infected humans undergoing hospitalization at the time t
$R_h(t)$	The number of humans who recovered in time t

Table 2 *Description of Covid-19 Mathematical Model Parameters*

Parameter	Information
Λ	Human Rate Recruitment
p	Human proportion of being vaccinated
μ_h	Pure human death rate
μ_c	Human death rate due to infection with covid-19
β	Proportion of contact between vulnerable humans and latent humans
α	Proportion of contact between latent humans and infected humans
η	Proportion of infected individuals undergoing hospitalization
γ_{h1}	The rate at which individual classes who treatment at the hospital experience recovery

γ_{h2}	The rate of infected class individuals recovering due to immune immunity
ε	Proportion of latent individuals undergoing treatment in hospital
$u_1(t)$	Proportion of susceptible humans who were vaccinated at the time of t with $0 \le u_1(t) \le 1$

Schematically the process of the covid-19 epidemic in humans can be presented in the transfer diagram in Figure 1.

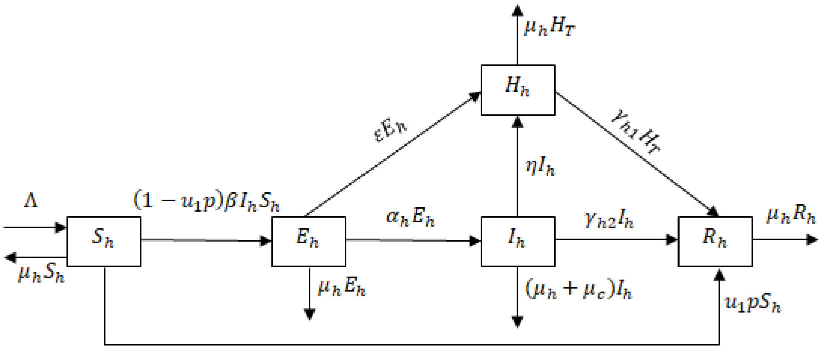


Figure 1. Covid-19 Epidemic Transfer Diagram in Humans

Based on Figure 1, the formulation of the covid-19 epidemic model in humans which is a system of differential equations as follows:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda - (1 - u_1 p)\beta I_h S_h - u_1 p S_h - \mu_h S_h \\
\frac{dE_h}{dt} &= \left(1 - u_{1p}\right)\beta I_h S_h - \alpha E_h - \varepsilon E_h - \mu_h E_h \\
\frac{dI_h}{dt} &= \alpha E_h - (\mu_h + \mu_c + \eta + \gamma_{h2}) I_h \\
\frac{dH_T}{dt} &= \varepsilon E_h + \eta I_h - (\mu_h + \gamma_{h1}) H_T \\
\frac{dR_h}{dt} &= u_1 p S_h + \gamma_{h1} H_T + \gamma_{h2} I_h - \mu_h R_h
\end{aligned} \tag{1.1}$$

Previously, we will analyze the uncontrolled model with $u_1(t) = 0$, so the system (1.1) defined as follows:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda - (\beta I_h + \mu_h) S_h \\
\frac{dE_h}{dt} &= \beta I_h S_h - (\mu_h + \alpha + \varepsilon) E_h \\
\frac{dI_h}{dt} &= \alpha E_h - (\mu_h + \mu_c + \eta + \gamma_{h2}) I_h \\
\frac{dH_T}{dt} &= \varepsilon E_h + \eta I_h - (\mu_h + \gamma_{h1}) H_T \\
\frac{dR_h}{dt} &= \gamma_{h1} H_T + \gamma_{h2} I_h - \mu_h R_h
\end{aligned} \tag{1.2}$$

From (1.2) We would find the equilibrium point by making zero on the right-hand side of the system. Define:

$$X = \mu_h + \alpha + \varepsilon,$$
$$Y = \mu_h + \mu_c + \eta + \gamma_{h2},$$
$$Z = \mu_h + \gamma_{h1}$$

So that:

$$\frac{dS_h}{dt} = \Lambda - (\beta I_h + \mu_h)S_h \tag{i}$$

$$\frac{dE_h}{dt} = \beta I_h S_h - XE_h \tag{ii}$$

$$\frac{dI_h}{dt} = \alpha E_h - YI_h \tag{iii}$$

$$\frac{dH_T}{dt} = \varepsilon E_h + \eta I_h - ZH_T \tag{v}$$

$$\frac{dR_h}{dt} = \gamma_{h1}H_T + \gamma_{h2}I_h - \mu_h R_h \tag{iv}$$

From (i) be obtained:

$$\Lambda - (\beta I_h + \mu_h)S_h = 0 \Leftrightarrow S_h = \frac{\Lambda}{\beta I_h + \mu_h} \tag{1}$$

From (iii) be obtained:

$$\alpha E_h - YI_h = 0 \Leftrightarrow E_h = \frac{YI_h}{\alpha} \tag{2}$$

Substitution (2) to equation (ii) be obtained:

$$\beta I_h S_h - \frac{XYI_h}{\alpha} = 0 \Leftrightarrow I_h = 0 \; atau \; S_h = \frac{XY}{\beta\alpha} \tag{3}$$

Case $I_h = 0$:

From (1) be obtained $S_h = \frac{\Lambda}{\mu_h}$ (4)

From (2) be obtained $E_h = 0$ (5)

From (5) and with substitution $I_h = 0$ and $E_h = 0$ to (iii) be obtained $H_T = 0$ (6)

So that equation (v) become:

$$R_h = 0 \tag{7}$$

Thus the diseases-free equilibrium point is as follows:

$$P_0 = \left(\frac{\Lambda}{\mu_h}, 0,0,0,0\right) \tag{1.3}$$

Case $I_h \neq 0$:

In the same way as before and assuming the endemic points are $P_1 = (S_h^*, E_h^*, I_h^*, H_T^*, R_h^*)$ then be obtained:

$$\begin{aligned} S_h^* &= \frac{XY}{\beta\alpha}, \\ E_h^* &= \frac{\Lambda\beta\alpha - \mu_h XY}{\beta\alpha X}, \\ I_h^* &= \frac{\Lambda\beta\alpha - \mu_h XY}{\beta XY}, \\ H_T^* &= \frac{(\Lambda\beta\alpha - \mu_h XY)(\gamma_{h2}\gamma_{h1}Y + \eta\alpha\gamma_{h1})}{\beta\alpha XYZ}, \\ R_h^* &= \frac{(\Lambda\beta\alpha - \mu_h XY)(\gamma_{h2}\gamma_{h1}Y + \eta\alpha\gamma_{h1})}{\mu_h\beta\alpha XYZ} \end{aligned} \tag{1.4}$$

A requirement that in the population there will always be individuals infected with Covid 19 if the value of $I_h^* > 0$.

To determine the basic reproduction number, it is assumed that $I_h^* > 0$.

We have $I_h^* = \frac{\Lambda\beta\alpha - \mu_h XY}{\beta XY}$

So notice that:

$$\begin{aligned} & I_h^* > 0 \\ \Leftrightarrow & \frac{\Lambda\beta\alpha - \mu_h XY}{\beta XY} > 0 \\ \Leftrightarrow & \Lambda\beta\alpha - \mu_h XY > 0 \end{aligned}$$

$\Leftrightarrow \dfrac{\Lambda\beta\alpha}{\mu_h XY} > 1$

So that it can defined the basic reproduction ratio number $R_0 = \frac{\Lambda\beta X}{\mu_h XY}$.

So we can get a more concise endemic point as follows:

$$\begin{aligned} S_h^* &= \frac{XY}{\beta\alpha}, \\ E_h^* &= \frac{\mu_h Y}{\beta\alpha}(R_0 - 1), \\ I_h^* &= \frac{\mu_h}{\beta}(R_0 - 1), \\ H_T^* &= \frac{(\varepsilon Y + \eta\alpha)\mu_h}{\beta\alpha Z}(R_0 - 1), \\ R_h^* &= \frac{(\gamma_{h2}\alpha Z + \varepsilon\gamma_{h1} Y + \eta\alpha\gamma_{h1})}{\beta\alpha Z}(R_0 - 1) \end{aligned} \tag{4.5}$$

Stability analysis is obtained based on the eigenvalues of the jacobian matrix through the linearization method:

$$J = \begin{pmatrix} -\beta I_h - \mu_h & 0 & -\beta S_h & 0 & 0 \\ \beta I_h & -X & \beta S_h & 0 & 0 \\ 0 & \alpha & -Y & 0 & 0 \\ 0 & \varepsilon & \eta & -Z & 0 \\ 0 & 0 & \gamma_{h2} & \gamma_{h1} & -\mu_h \end{pmatrix}$$

For the disease-free point then

$$J(P_0) = \begin{pmatrix} -\mu_h & 0 & -\dfrac{\beta\Lambda}{\mu_h} & 0 & 0 \\ 0 & -X & \dfrac{\beta\Lambda}{\mu_h} & 0 & 0 \\ 0 & \alpha & -Y & 0 & 0 \\ 0 & \varepsilon & \eta & -Z & 0 \\ 0 & 0 & \gamma_{h2} & \gamma_{h1} & -\mu_h \end{pmatrix}$$

So that $\det\big(\lambda I - J(P_0)\big) = 0$

$$(\lambda + \mu_h)\big(\lambda_\mu + \mu_h\big)\begin{vmatrix} \lambda + X & -\dfrac{\beta\Lambda}{\mu_h} & 0 \\ -\alpha & \lambda + Y & 0 \\ -\varepsilon & -\eta & \lambda + Z \end{vmatrix} = 0$$

$$\Leftrightarrow (\lambda + \mu_h)(\lambda + \mu_h)(\lambda + Z)\begin{vmatrix} \lambda + X & -\dfrac{\beta\Lambda}{\mu_h} \\ -\alpha & \lambda + Y \end{vmatrix} = 0$$

$$\Leftrightarrow (\lambda + \mu_h)(\lambda + \mu_h)(\lambda + Z)\left[\lambda^2 + (X + Y)\lambda + XY - \frac{\beta\Lambda\alpha}{\mu_h}\right] = 0$$

So that the eigenvalues are obtained:

$\lambda_1 = -\mu_h, \lambda_2 = -\mu_h, \lambda_3 = -Z$ and consider the following equation:

$$a_0\lambda^2 + a_1\lambda^2 + a_2 = 0 \tag{I}$$

With $a_0 = 1, a_1 = X + Y, a_2 = XY - \frac{\beta\Lambda\alpha}{\mu_h} = XY(1 - R_0)$.

Clearly $a_2 > 0$ if $R_0 < 1$ and $a_2 < 0$ if $R_0 > 1$.

Clearly $\lambda_{4,5} = \frac{-a_1 \pm \sqrt{D}}{2a_0}$.

From (I) will be shown $D > 0$.

Case $R_0 < 1$.

Clearly

$D = a_1^2 - 4a_0a_2$

$= (X + Y)^2 - 4XY(1 - R_0)$

$= X^2 + Y^2 + 2XY - 4XY + 4R_0$
$= X^2 + Y^2 - 2XY + 4R_0$
$= (X - Y)^2 + 4R_0 > 0.$
Case $R_0 > 1$:
Clearly $a_2 < 0$ caused $D > 0$.
Will be shown $-a_1 + \sqrt{D} < 0$.
Note that
$0 < a_1^2 - 4a_2 < a_1^2$
$\Leftrightarrow 0 < \sqrt{D} < a_1$
$\Leftrightarrow -a_1 < -\sqrt{D} < 0$
$\Leftrightarrow -a_1 + \sqrt{D} < 0.$
Because of $-a_1 + \sqrt{D} < 0$ then the eigenvalues $\lambda_{4,5}$ is a negative number.
To an endemic point $P_1 = (S_h^*, E_h^*, I_h^*, H_T^*, R_h^*)$:

$$J(P_1) = \begin{pmatrix} -\beta I_h^* - \mu_h & 0 & -\beta S_h^* & 0 & 0 \\ \beta I_h^* & -X & \beta S_h^* & 0 & 0 \\ 0 & \alpha & -Y & 0 & 0 \\ 0 & \varepsilon & \eta & -Z & 0 \\ 0 & 0 & \gamma_{h2} & \gamma_{h1} & -\mu_h \end{pmatrix}$$

In the same way as before that is by completing $\det\big(\lambda I - J(P_1)\big) = 0$ then the characteristic equation can be obtained:
$(\lambda + \mu_h)(\lambda + Z)[\lambda^3 + (\mu_h R_0 + X + Y)\lambda^2 + [(X + Y)\mu_h R_0]\lambda + \mu_h XY(R_0 - 1)] = 0$
So that it is obtained
$\lambda_1 = -\mu_h, \lambda_2 = -2$ and consider the following equation:
$x_0\lambda^3 + x_1\lambda^2 + x_2\lambda + x_3 = 0$
With $x_0 = 1, x_1 = \mu_h R_0 + X + Y, \quad x_2 = (X + Y)\mu_h R_0, \; x_3 = \mu_h XY(R_0 - 1)$ (II)
To check the roots of the equation (II) has a real negative part would be used by the Routh-Hurwitz Criterion, namely by showing:
$i. x_1 > 0, x_2 > 0, x_3 > 0$
$ii. x_1 x_2 - x_3 > 0$
Let endemic equilibrium point there is a time $R_0 > 1$
So value $R_0 - 1 > 0$,
Thus it is obtained that $x_1, x_2, x_3 > 0$
Then notice that:
$x_1 x_2 - x_3 = (\mu_h R_0 + X + Y)\big((X + Y)\mu_h R_0\big) - \big(\mu_h XY(R_0 - 1)\big)$
$= \mu_h X^2 R_0^2 + \mu_h X^2 R_0 + \mu_h XY R_0 + \mu_h^2 Y R_0^2 + \mu_h Y^2 R_0 + \mu_h XY$
So value $x_1 x_2 - x_3 > 0$ if $R_0 > 1$.
Thus, the value of λ_1, λ_2 and the real part $\lambda_3, \lambda_4, \lambda_5$ negative value. So point P_1 is said to be locally asymptotically stable if $R_0 > 1$.

2.2 Parameter Estimation

This chapter discusses the formulation of a mathematical model of Covid-19 by dividing the human popula

In this section, we will describe the parameter estimation in the dengue model using the maximum likelihood method. The following is the construction of the CTMC model (*Continous Time Markov Chain*) for the established Covid-19 epidemic model by reviewing only the population that can cause the spread of the Covid-19 epidemic:

$$P\Delta t=\begin{cases}\beta I_h S_h \Delta t & ,(s_h-1,e_h+1,i_h,h_t)\\ \alpha_h E_h \Delta \mathrm{t} & ,(\mathrm{s_h},e_h-1,i_h+1,h_t)\\ \varepsilon \mathrm{E_h}\Delta t & ,(s_h,e_h-1,i_h,h_t+1)\\ \eta I_h \Delta t & ,(s_h,e_h,i_h-1,h_t+1)\\ (\gamma_{h1}+\gamma_{h2})H_T\Delta t & ,\quad (s_h,e_h,i_h,h_t+1)\\ 1-(\beta I_h S_h+\alpha_h E_h+\varepsilon E_h+\eta I_h+(\gamma_{h1}+\gamma_{h2})H_T)\Delta t\,, & (s_h,e_h,i_h,h_t)\end{cases}$$

$$\begin{aligned}L(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})\\ =\int_0^T \beta I_h(t)S_h(t)dt\int_0^T \alpha_h E_h(t)dt\int_0^T \varepsilon E_h(t)dt\int_0^T \eta I_h(t)dt\int_0^T(\gamma_{h1}+\gamma_{h2})H_T\,dt+\int_0^T X1\,dt\\ +\exp\left(-\int_0^T (X2+X1)dt\right)\end{aligned}$$

with

$X_1=1-(\beta I_h(t)S_h(t)+\alpha_h E_h(t)+\varepsilon E_h(t)+\eta I_h(t)+(\gamma_{h1}+\gamma_{h2})H_T)$,
$X_2=\beta I_h(t)S_h(t)+\alpha_h E_h(t)+\varepsilon E_h(t)+\eta I_h(t)+(\gamma_{h1}+\gamma_{h2})H_T$.

Note that:

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial\beta}=0$$

$$\begin{aligned}\Leftrightarrow\left(\frac{\int_0^T I_h(t)S_h(t)dt}{\int_0^T \beta I_h(t)S_h(t)dt}-\int_0^T I_h(t)S_h(t)dt\right)\\ +\left(\frac{-\int_0^T I_h(t)S_h(t)dt}{\int_0^T\big(1-(\beta I_h(t)S_h(t)+\alpha_h E_h(t)+\varepsilon E_h(t)+\eta I_h(t)+(\gamma_{h1}+\gamma_{h2})H_T)\big)dt}\right)-\int_0^T -I_h(t)S_h(t)dt\\ =0\end{aligned}$$

$$\Leftrightarrow\frac{1}{\beta}=\frac{\int_0^T I_h(t)S_h(t)dt}{\int_0^T\big(1-(\beta I_h(t)S_h(t)+\alpha_h E_h(t)+\varepsilon E_h(t)+\eta I_h(t)+(\gamma_{h1}+\gamma_{h2})H_T)\big)dt}$$

$$\Leftrightarrow 2\beta\int_0^T I_h(t)S_h(t)dt=T-\alpha_h\int_0^T E_h(t)dt-\varepsilon\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)\,dt-(\gamma_{h1}+\gamma_{h2})\int_0^T H_T dt \qquad (1^*)$$

In the same way can be obtained:

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial\alpha_h}=0$$

$$\Leftrightarrow 2\alpha_h\int_0^T E_h(t)dt=T-\beta\int_0^T I_h(t)S_h(t)dt-\varepsilon\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)\,dt-(\gamma_{h1}+\gamma_{h2})\int_0^T H_T dt \qquad (2^*)$$

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial\varepsilon}=0$$

$$\Leftrightarrow 2\varepsilon\int_0^T E_h(t)dt=T-\beta\int_0^T I_h(t)S_h(t)dt-\alpha_h\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)\,dt-(\gamma_{h1}+\gamma_{h2})\int_0^T H_T dt \qquad (3^*)$$

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial\eta}=0$$

$$\Leftrightarrow 2\eta\int_0^T I_h(t)dt=T-\beta\int_0^T I_h(t)S_h(t)dt-\alpha_h\int_0^T E_h(t)dt-\varepsilon\int_0^T E_h(t)\,dt-(\gamma_{h1}+\gamma_{h2})\int_0^T H_T dt \qquad (4^*)$$

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial(\gamma_{h1}+\gamma_{h2})}=0$$

$$\Leftrightarrow 2(\gamma_{h1}+\gamma_{h2})\int_0^T H_T(t)dt=T-\beta\int_0^T I_h(t)S_h(t)dt-\alpha_h\int_0^T E_h(t)dt-\varepsilon\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)\,dt \qquad (5^*)$$

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial\gamma_{h1}}=0$$

$$\Leftrightarrow 2\gamma_{h1}\int_0^T H_T(t)dt=T-\beta\int_0^T I_h(t)S_h(t)dt-\alpha_h\int_0^T E_h(t)dt-\varepsilon\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)\,dt-\gamma_{h2}\int_0^T H_T(t)\,dt \qquad (6^*)$$

$$\frac{\partial lnL(\beta,\alpha_h,\varepsilon,\eta,\gamma_{h1},\gamma_{h2})}{\partial\gamma_{h2}}=0$$

$$\Leftrightarrow 2\gamma_{h2}\int_0^T H_T(t)dt=T-\beta\int_0^T I_h(t)S_h(t)dt-\alpha_h\int_0^T E_h(t)dt-\varepsilon\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)\,dt-\gamma_{h1}\int_0^T H_T(t)\,dt \qquad (7^*)$$

Substitution (3*) to (2*) can be obtained:

$$2\alpha_h\int_0^T E_h(t)dt=\frac{T-\beta\int_0^T I_h(t)S_h(t)dt+\alpha_h\int_0^T E_h(t)dt-\eta\int_0^T I_h(t)dt-(\gamma_{h1}+\gamma_{h2})\int_0^T H_T(t)dt}{2}$$

$$\Leftrightarrow 3\alpha_h \int_0^T E_h(t)dt = T - \beta \int_0^T I_h(t)S_h(t)dt - \eta \int_0^T I_h(t)dt - (\gamma_{h1} + \gamma_{h2}) \int_0^T H_T(t)dt \qquad (8^*)$$

From (8*) and (2*) can be conclude that $\alpha_h = \varepsilon$.

Substitution (4*) to (1*) can be obtained:

$$2\beta \int_0^T I_h(t)S_h\,(t)dt = \frac{T}{2} - \alpha_h \int_0^T E_h(t)dt + \frac{\beta \int_0^T I_h(t)S_h(t)dt}{2} - \frac{(\gamma_{h1} + \gamma_{h2}) \int_0^T H_T(t)dt}{2}$$

$$\Leftrightarrow \frac{3}{2}\beta \int_0^T I_h(t)S_h\,(t)dt = \frac{T}{2} - \alpha_h \int_0^T E_h(t)dt - \frac{(\gamma_{h1}+\gamma_{h2}) \int_0^T H_T(t)dt}{2} \qquad (9^*)$$

Substitution (4*) to (9*) can be obtained:

$$2(\gamma_{h1} + \gamma_{h2}) \int_0^T H_T(t)dt = \frac{T}{2} - \frac{\beta \int_0^T I_h(t)S_h(t)dt}{2} - \alpha_h \int_0^T E_h(t)\,dt + \frac{(\gamma_{h1}+\gamma_{h2}) \int_0^T H_T(t)dt}{2}$$

$$\Leftrightarrow \frac{3}{2}(\gamma_{h1} + \gamma_{h2}) \int_0^T H_T(t)dt = \frac{T}{2} - \frac{\beta \int_0^T I_h(t)S_h(t)dt}{2} - \alpha_h \int_0^T E_h(t)\,dt \qquad (10^*)$$

Substitution (10*) to (9*) can be obtained:

$$\frac{3}{2}\beta \int_0^T I_h(t)S_h(t)dt = \frac{T}{2} - \alpha_h \int_0^T E_h(t)dt - \frac{T}{6} - \frac{\beta \int_0^T I_h(t)S_h(t)dt}{6} - \frac{\alpha_h \int_0^T E_h(t)dt}{3}$$

$$\Leftrightarrow \frac{5}{3}\beta \int_0^T I_h(t)S_h(t)dt = \frac{T}{3} - \frac{4\alpha_h \int_0^T E_h(t)dt}{3}$$

$$\Leftrightarrow \beta = \frac{T-4\alpha_h \int_0^T E_h(t)dt}{5 \int_0^T I_h(t)S_h(t)dt} \qquad (11^*)$$

Substitution (10*) to (4*) can be obtained:

$$\eta = \frac{2T}{3} - \frac{2\beta \int_0^T I_h(t)S_h(t)dt}{3} - \frac{4\alpha_h \int_0^T E_h(t)}{3} \qquad (12^*)$$

Substitution (4*), (9*) to (1*) can be obtained:

$$3\alpha_h \int_0^T E_h(t)dt = T - \beta \int_0^T I_h(t)S_h(t)dt - \left[\frac{2T}{6} - \frac{2\beta \int_0^T I_h(t)S_h(t)dt}{6} - \frac{4\alpha \int_0^T E_h(t)dt}{6}\right] - \left[\frac{T}{3} - \frac{\beta \int_0^T I_h(t)S_h(t)dt}{3} - \frac{2\alpha_h \int_0^T E_h(t)dt}{3}\right]$$

$$\Leftrightarrow \frac{5}{3}\alpha_h \int_0^T E_h(t)dt = \frac{T}{3} - \frac{\beta \int_0^T I_h(t)S_h(t)dt}{3}$$

$$\Leftrightarrow \alpha_h = \frac{T-\beta \int_0^T I_h(t)S_h(t)dt}{5 \int_0^T E_h(t)dt} \qquad (13^*)$$

Substitution (11*) to (13*) can be obtained:

$$\alpha_h = \frac{T-\frac{T}{5}+\frac{4}{5}\alpha_h \int_0^T E_h(t)dt}{\int_0^T E_h(t)dt}$$

$$\Leftrightarrow \frac{21\alpha_h \int_0^T E_h(t)dt}{5} = \frac{4T}{5}$$

$$\Leftrightarrow \alpha_h = \frac{4T}{21 \int_0^T E_h(t)dt} \qquad (14^*)$$

Substitution (14*) to (11*) can be obtained:

$$\beta = \frac{T-\frac{16T}{21}}{5 \int_0^T I_h(t)S_h(t)dt} = \frac{T}{21 \int_0^T I_h(t)S_h(t)dt} \qquad (15^*)$$

Substitution (14*), (15*) to (12*) can be obtained:

$$2\eta \int_0^T I_h(t)dt = \frac{2T}{3} - \frac{2\left[\frac{T}{21 \int_0^T I_h(t)S_h(t)dt}\right] \int_0^T I_h(t)S_h(t)dt}{3} - \frac{4\left[\frac{4T}{21 \int_0^T E_h(t)dt}\right] \int_0^T E_h(t)dt}{3} \qquad (16^*)$$

$$\Leftrightarrow \eta = \frac{\frac{2T}{3} - \frac{2T}{63} - \frac{16T}{63}}{2 \int_0^T I_h(t)dt} = \frac{24T}{126 \int_0^T I_h(t)dt}$$

Substitution (14*), (15*) and (16*) to (6*) can be obtained:

$$\gamma_{h2}\int_0^T H_T(t)dt = \frac{54T}{252} - \frac{\gamma_{h1}\int_0^T H_T(t)dt}{2} \tag{17*}$$

Substitution (8*) to (7*) can be obtained:

$$2\gamma_{h1}\int_0^T H_T(t)dt = \frac{54T}{126} - \frac{54T}{252} + \frac{\gamma_{h1}\int_0^T H_T(t)dt}{2}$$
$$\Leftrightarrow \frac{3}{2}\gamma_{h1}\int_0^T H_T(t)dt = \frac{54T}{252}$$
$$\Leftrightarrow \gamma_{h1} = \frac{3T}{21\int_0^T H_T(t)dt} \tag{18*}$$

Substitution (18*) to (17*) can be obtained:

$$\gamma_{h2}\int_0^T H_T(t)dt = \frac{54T}{252} - \frac{3T}{42}$$
$$\Leftrightarrow \gamma_{h2} = \frac{3T}{21\int_0^T H_T(t)dt} \tag{19*}$$

So the parameter estimates obtained are:

$$\beta = \frac{T}{21\int_0^T I_h(t)S_h(t)dt};\ \alpha_h = \frac{4T}{21\int_0^T E_h(t)dt};\varepsilon = \frac{4T}{21\int_0^T E_h(t)dt};\eta = \frac{24T}{126\int_0^T I_h(t)dt};\gamma_{h1} = \frac{3T}{21\int_0^T H_T(t)dt};\gamma_{h2} = \frac{3T}{21\int_0^T H_T(t)dt}$$

So that the estimated parameters used in the COVID-19 epidemic model are presented in the following Table 3.

Table 3 ***Covid-19 Mathematical Model Parameter Values***

Parameter	Value	Parameter	Value
μ_h	0.0167	η	0.132
μ_c	0.3	γ_{h1}	0.323
β	0.0732	γ_{h2}	0.165
α	0.05464	ε	0.041

3. Results and Discussion

In this section, we will solve the problem of optimal control in the covid 19 epidemic model by vaccination. The optimal control problem in a system of equations with control consists of the objective features given as follows:

$$J = min\int_0^{t_f}(I_h + B_1u_1^2)dt$$

With $0 \le t \le t_f$, $0 \le u_1(t) \le 1$, t_f is final time, I_h number of people infected with covid 19, B_1 is a positive constant representing the vaccination weight.

$$H = I_h + B_1u_1^2 + \lambda[\lambda - (1-u_1p)\beta I_hS_h - u_1pS_h - \mu_hS_h] + \lambda_5[u_1pS_h + \gamma_{h1}H_T + \gamma_{h2}I_h - \mu_hR_h] +$$
$$+\lambda_2[(1-u_1p)\beta I_hS_h - \alpha E_h - \varepsilon E_h - \mu_hE_h] + \lambda_3[\alpha E_h - (\mu_h + \mu_c + \eta + \gamma_{h2})I_h$$
$$+\lambda_4[\varepsilon E_h + \eta I_h - (\mu_h + \gamma_{h1})H_T]$$

The Hamiltonian function will reach an optimal solution if it satisfies the conditions of the state equation, costate and stationary conditions.

State equation:

$$\dot{x} = \frac{\partial H}{\partial \lambda}$$

Costate equation:

$$\lambda_1 = -\frac{\partial H}{\partial S_h} = (1-u_1p)\beta I_h(\lambda_1 - \lambda_2) + u_1p(\lambda_1 - \lambda_5) + \lambda_1\mu_h$$
$$\lambda_2 = -\frac{\partial H}{\partial E_h} = \alpha(\lambda_2 - \lambda_3) + \varepsilon(\lambda_2 - \lambda_4) + \lambda_2\mu_h$$

$$\lambda_3 = -\frac{\partial H}{\partial I_h} = (1 - u_1 p)\beta S_h(\lambda_1 - \lambda_2) + \eta(\lambda_3 - \lambda_4) + \gamma_{h2}(\lambda_3 - \lambda_5) + (\mu_h + \mu_c)\lambda_3$$
$$\lambda_4 = \gamma_{h1}(\lambda_4 - \lambda_5) + \lambda_4\mu_h$$
$$\lambda_5 = -\frac{\partial H}{\partial R_h} = \lambda_5\mu_h$$

Stationary Conditions:

$$\frac{\partial H}{\partial u_1} = 0$$

So can be obtained $u_1 = \frac{p\beta I_h S_h(\lambda_2 - \lambda_1) + pS_h(\lambda_1 - \lambda_5)}{2B_1}$

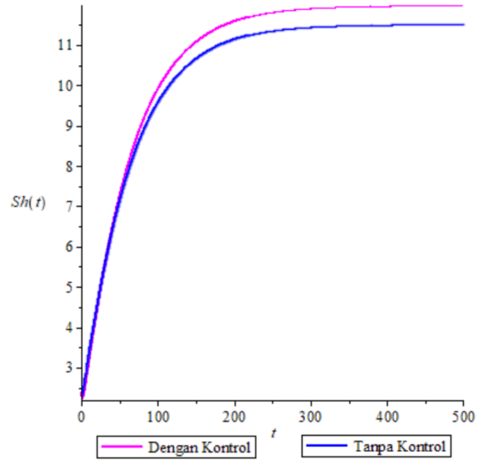
Thus the optimal control value can be expressed as follows:

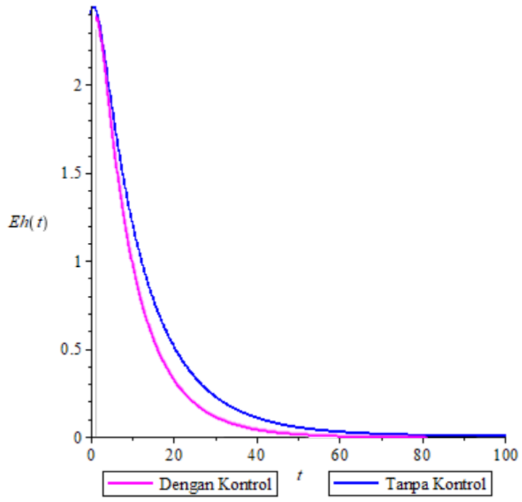
$$u_1^*(t) = \min\left(1, max\left(0, \frac{p\beta I_h S_h(\lambda_2 - \lambda_1) + pS_h(\lambda_1 - \lambda_5)}{2B_1}\right)\right).$$

The optimal control problem with Pontryagin Minimum Principle can be solved numerically, by designing an algorithm that produces an approximation of the optimal control value. Note that the optimal system consists of an equation of state with initial conditions, an equation of state with a transversality condition, and the characteristics of the optimal control. This optimal control problem can be solved by a backward-forward sweep algorithm as follows:

1. Time interval $[0, t_f]$ divided into N subinterval uniform. Vector $\vec{x} = (x_1, x_2, \dots, x_5)$ and vector $\vec{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_5)$ which is an approximation vector for state and co-state.
2. Make an initial guess for $\overrightarrow{\overline{u_1}}$ over the time interval, with $\overrightarrow{\overline{u_1}}(0) = 0$ based on the state differential equation on the optimal system with the order 4 Runge-Kutta method. $\overrightarrow{\overline{u_1}}(t)$ is a control approximation u_1 at the time t.
3. With the initial state $x_1 = x(t_0) = x_0$ and value on $\overrightarrow{\overline{u_1}}$, would be done the solution of state value $(\vec{x})$ by forward at the time t. Based on the differential co-state on the optimal system with the order 4 Runge-Kutta method.
4. With the condition of transversality $\lambda_{N+1} = \lambda(t_f) = 0$ and value $\overrightarrow{\overline{u_1}}$ can be solved $\vec{\lambda}$ by at the time t.
5. Update value $\overrightarrow{\overline{u_1}}$ by substituting the value of $\vec{x}$ and $\vec{\lambda}$ into the optimal control characterization.
6. Check for convergence. If the variable values in the current and previous iterations are close enough, then the current value is the solution. If not, then go back to step 2.

The simulation results are presented as follows:





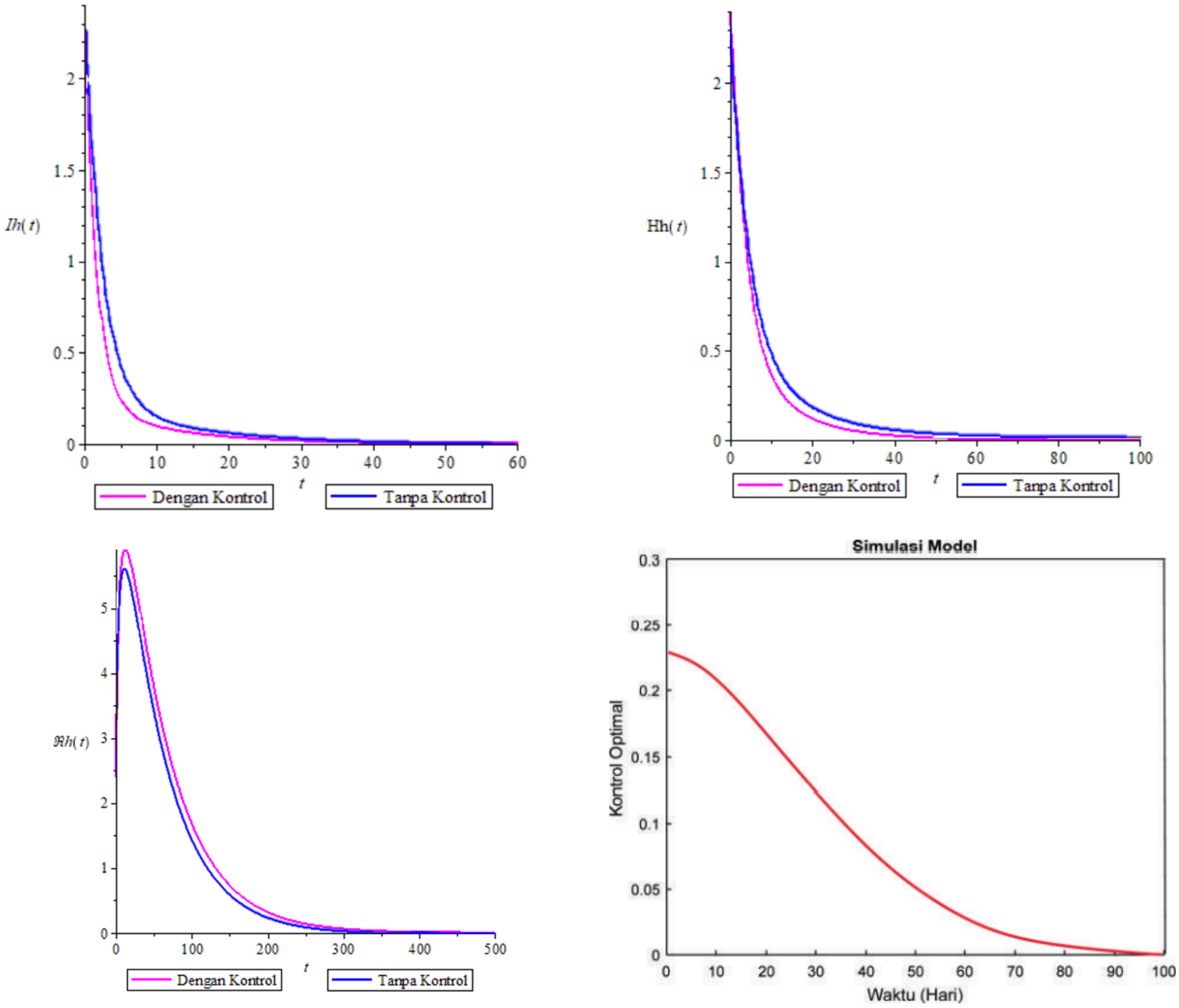


Figure 2. Population dynamics of the covid19 model with the vaccination program

Based on Figure 1. that means at first the optimal control value u_1^* given with a sufficiently maximal effort, then control the optimal value slowly along with the decrease in infected individuals. Based on the picture, it can also be seen that the size of the infected human population has increased slightly, this is due to the addition of a graph of the vulnerable human population infected with COVID-19. Then the graph of the size of the human population that has decreased, this is caused by the reduction of the human population caused by death or caused by humans who have recovered from infection due to the treatment process or the individual's strong immunity.

4. Conclusion

This study formulated and analyzed a compartmental mathematical model of Covid-19 transmission incorporating susceptible, exposed, infected, hospitalized, and recovered populations under realistic epidemiological assumptions. The analysis showed that the disease-free equilibrium is locally asymptotically stable when the basic reproduction number $R_0 < 1$, while an endemic equilibrium exists and is stable when $R_0 > 1$, indicating sustained transmission. Model parameters were estimated using the maximum likelihood method within a continuous-time Markov chain framework to better capture transmission dynamics. Furthermore, an optimal vaccination control strategy was derived using the Pontryagin Minimum Principle and solved numerically via the backward–forward sweep algorithm. Simulation results indicate that strong early vaccination efforts substantially reduce infection levels, after which control intensity can be gradually decreased, demonstrating that timely and optimally designed vaccination policies are effective in minimizing the spread and overall impact of Covid-19.

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